Section 4.4

The Fundamental Theorem of Calculus: If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

1) Evaluate each definite integral.

a) $\int_0^2 (2x^3 + 1)\,dx$

b) $\int_1^3 \frac{x-1}{\sqrt{x}}\,dx$

c) $\int_0^{\pi/6} \sec x \tan x\,dx$

d) $\int_0^4 |2x - 4|\,dx$

2) Find the area of the region bounded by the graph of $y = 3x^2 - 2x + 1$, the x-axis, the y-axis, and the vertical line $x = 2$.

The Mean Value Theorem for Integrals: If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

Definition of the Average Value of a Function on an Interval: If f is integrable on a closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

3) Find the average value of $f(x) = 9 - x^2$ on the interval $[-3, 3]$.

4) Evaluate the function $F(x) = \int_0^x t^2\,dt$ at $x = 0, 1, 2,$ and 3.

The Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

5) Evaluate $\frac{d}{dx}\left[\int_1^x \sqrt[3]{t^3+1}\,dt\right]$.

6) Find the derivative of $F(x) = \int_0^{x^3} \sin t^2\,dt$.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #22, 27, 37, 41, 48, 55, 70, 77, 83, 89